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Procedure for Probabilistic Tsunami Hazard Assessment for Incomplete and Uncertain Data

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Fundamental Question

How to assess the likelihood of such an event??

Proposed Approach

- Empirical approach based on tsunami **observations** which are often **incomplete** and **uncertain**.

- Alternative to empirical approach is **tsunami propagation modelling.**

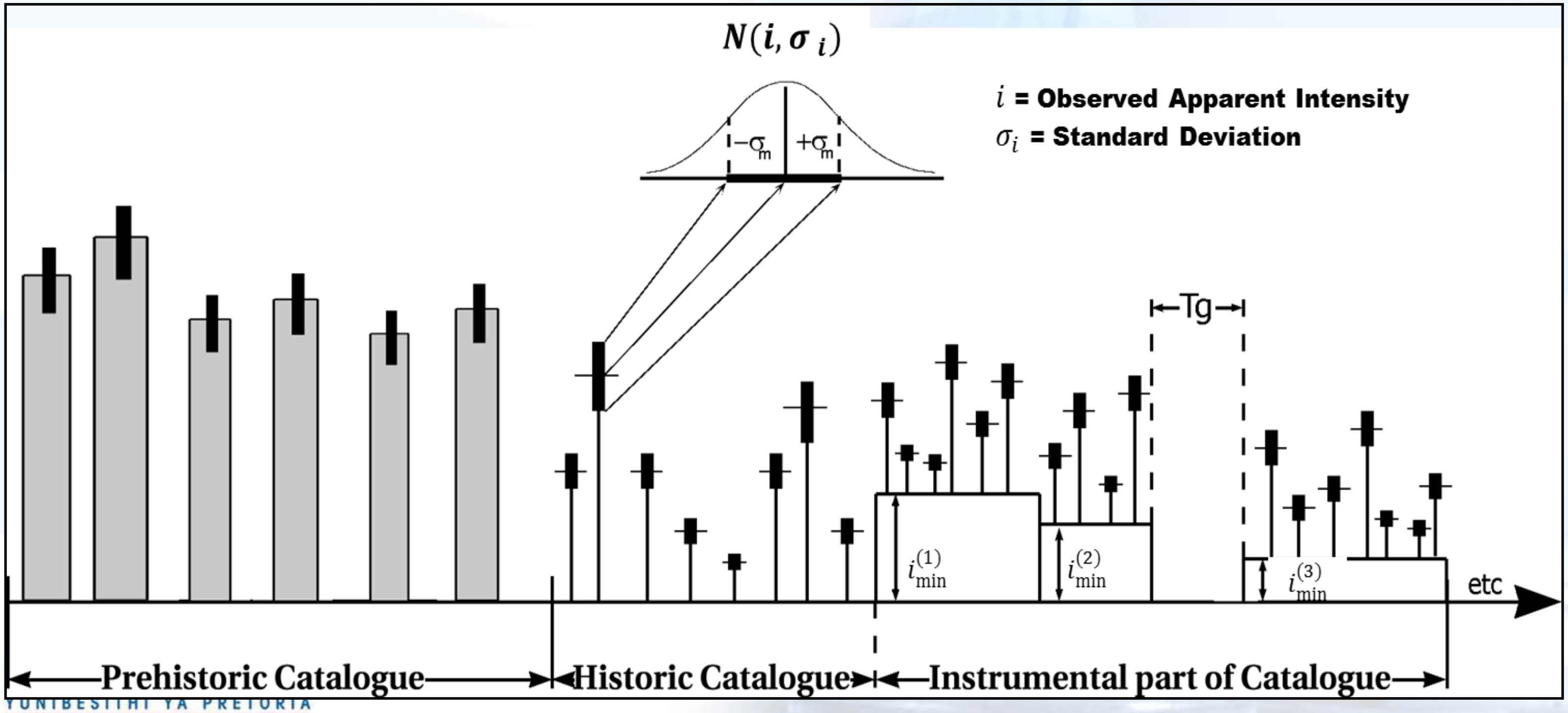
  (Comprehensive review of alternative approaches e.g. Geist, 2009)

Problems Faced Using Only Observations

1. **Incompleteness** of tsunami database.
2. Uncertainty in the **intensity** (magnitude/size) of an event.
3. Uncertainty in the applied **occurrence model**.
4. **Maximum** local/regional tsunami intensity.

Nature of Tsunami Input Data

- Very strong pre-historic events (palaeo) – last thousands of years.
- Strongest historic events – last few hundred years.
- Recent events – in last 100 years (complete catalogue).



Modified: Kijko & Sellevoll, 1992

Model Parameters

- Tsunami hazard described by probabilities and return periods is characterised by 3 parameters:

λ	Mean activity rate of tsunami occurrence
b-value	Ratio between strong and weak intensity tsunamis (equivalent to the frequency-magnitude G-R relation)
$i_{\max}$	Upper limit of tsunami intensity (or tsunami run-up)

Proposed PTHA Model

The proposed methodology addresses:

A. Temporal distribution.

B. Intensity distribution.

- Discrepancy between data and occurrence model.

C. Combination of Extreme and Complete catalogues.

A. Temporal Distribution of Tsunamis

Prob [n tsunami events, observed in time interval t along a coast]

$$p(n|\lambda,t)=\frac{(\lambda t)^n}{n!}\exp(-\lambda t)\quad n=0,1,\ldots$$

Poisson-Gamma **PDF**

$$p(n|\lambda,t)=\frac{\Gamma(n+q_\lambda)}{n!\,\Gamma(q_\lambda)}\left(\frac{p_\lambda}{t+p_\lambda}\right)^{q_\lambda}\left(\frac{t}{t+p_\lambda}\right)^{n}$$

where

- $p_\lambda=\bar{\lambda}/\sigma_\lambda^2$
- $q_\lambda=\bar{\lambda}p_\lambda$

B. Distribution of Tsunami Intensity

Assume

- Observed intensity i = unknown “true” intensity + random error
- Random error can be significant ~ Gaussian (0, SD)

(Tinti & Mulargia, 1985)

- Intensity of tsunami occurrence often very uncertain

…especially for palaeo and historic events

B. Distribution of Tsunami Intensity

Assume

Tsunami run-up heights (h in meters) follows negative exponential distribution similar to G-R relationship

❖ Soloviev (1969) introduced a frequency-size distribution

$$\lambda = a \times 10^{-bi}$$

were a and b are regression coefficients.

❖ Intensity i can be linked with tsunami run-up (h) along a coastline

$$i = log_2(\sqrt{2h})$$

B. Distribution of Tsunami Intensity

CDF of tsunami intensity

$$f_{\mathrm{I}}(i) = \begin{cases} 0 & \\ \dfrac{\beta \exp[-\beta(i - i_{\min})]}{1 - \exp[-\beta(i_{\max} - i_{\min})]} & \text{for} \quad i_{\min} \le i \le i_{\max} \\ 0 & \end{cases} \qquad \begin{matrix} i_{\min} < i \\ \\ i > i_{\max} \end{matrix}$$

Compound **CDF**

$$f_{\mathrm{I}}(i|i_{\min}) = \left[1 - \left(\frac{q_{\beta}}{q_{\beta} + \bar{\beta}(i_{\max} - i_{\min})}\right)^{q_{\beta}}\right]^{-1} \left[\bar{\beta}\left(\frac{q_{\beta}}{q_{\beta} + \bar{\beta}(i - i_{\min})}\right)^{q_{\beta}+1}\right]$$

where $q_{\beta} = \bar{\beta} p_{\beta}$

C. Combination of Catalogues

For each part of tsunami catalogues build likelihood function:

- $L_{\text{Paleo}}(\boldsymbol{\theta}) =$ likelihood function based on **palaeo** tsunamis
- $L_{\text{Historic}}(\boldsymbol{\theta}) =$ likelihood function based on **historic** tsunamis
- $L_{\text{Complete}}(\boldsymbol{\theta}) =$ likelihood function based on **complete** tsunami catalogues

Total likelihood function:

$$\boxed{L(\boldsymbol{\theta}) = L_{\text{Paleo}}(\boldsymbol{\theta}) \times L_{\text{Historic}}(\boldsymbol{\theta}) \times L_{\text{Complete}}(\boldsymbol{\theta})}$$

Parameter Estimation of λ and β

$$\boxed{L(\boldsymbol{\theta}) = L_{\text{Paleo}}(\boldsymbol{\theta}) \times L_{\text{Historic}}(\boldsymbol{\theta}) \times L_{\text{Complete}}(\boldsymbol{\theta})}$$

Obtain parameter estimates $\bar{\lambda}$ and $\bar{\beta}$ through the

Maximum Likelihood Estimation Method

which for given $i_{\max}$ maximizes the likelihood function $L(\boldsymbol{\theta})$.

Obtained by solving: $\begin{bmatrix} \dfrac{\partial \ln[L(\boldsymbol{\theta})]}{\partial \bar{\lambda}} \\ \\ \dfrac{\partial \ln[L(\boldsymbol{\theta})]}{\partial \bar{\beta}} \end{bmatrix} = 0$ where $\boldsymbol{\theta} = (\bar{\lambda}, \bar{\beta}, i_{\max})$

Parameter Estimation of $i_{\max}$

$$\boxed{i_{max} = i_{\max}^{obs} + \Delta}$$

where $\Delta = \dfrac{\delta^{1/q_\beta}\exp[nr^{q_\beta}/(1-r^{q_\beta})]}{\bar{\beta}}\left[\Gamma\left(-\dfrac{1}{q_\beta}, \delta r^{q_\beta}\right) - \Gamma\left(-\dfrac{1}{q_\beta}, \delta\right)\right]$

(Kijko, 2004; Kijko and Singh, 2011)

• $r = \dfrac{p_\beta}{p_\beta + i_{max} - i_{min}}$	• $C_\beta = \left[1 - \left(\dfrac{q_\beta}{q_\beta + \bar{\beta}(i_{\max} - i_{\min})}\right)^{q_\beta}\right]^{-1}$
• $\delta = nC_\beta$	• $p_\beta = \bar{\beta}/\sigma_\beta^2$ & $q_\beta = \left(\bar{\beta}/\sigma_\beta\right)^2$
$\Gamma(\cdot,\cdot)$ = complementary Incomplete Gamma Function (Abramowitz and Stegun, 1970)	

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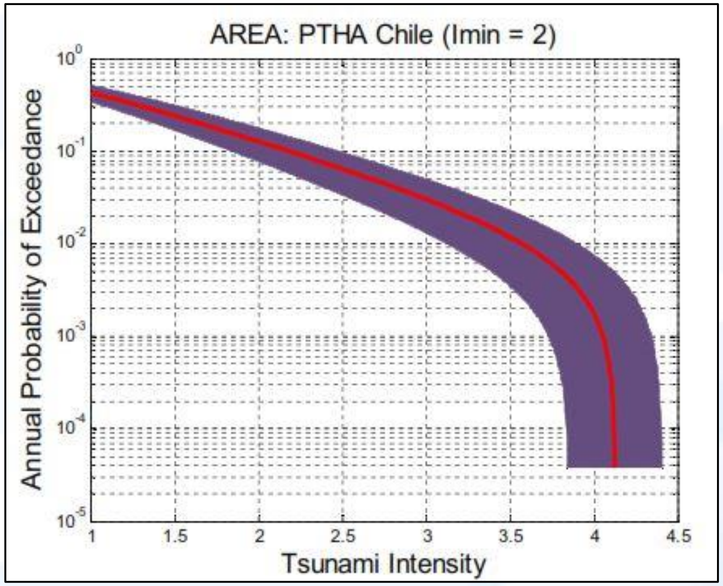
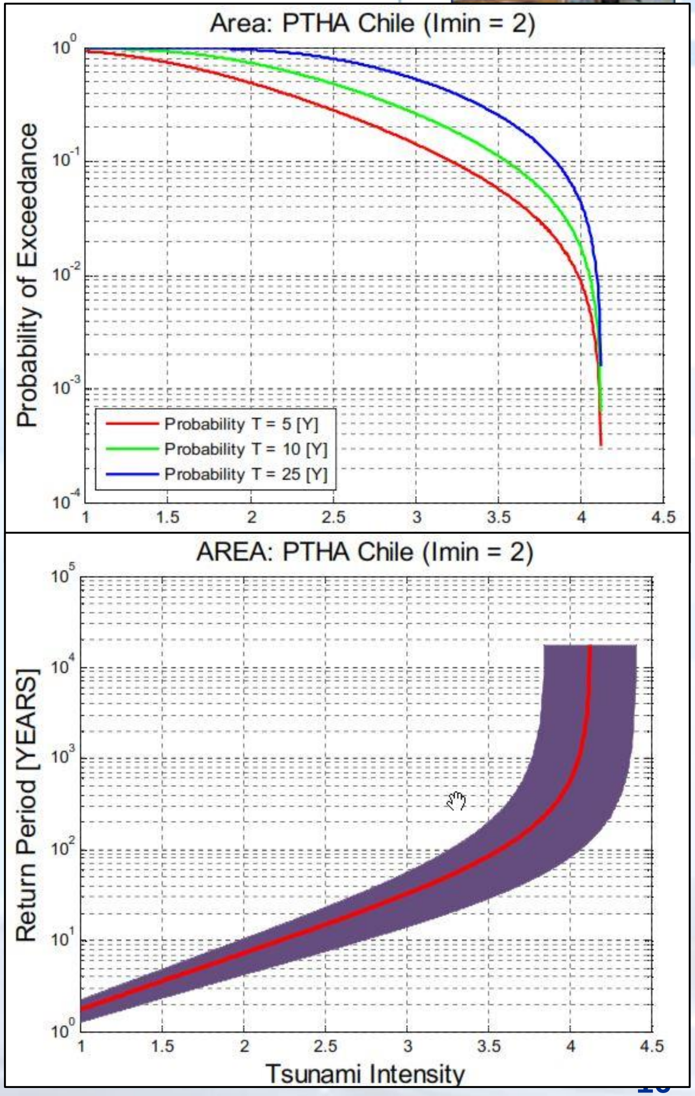
Example 1: Chilé

Only Extreme (Historic) events catalogues





Tsunami database of Dr V.K. Gusiakov,
Institute of Computational Mathematics and Geophysics, Siberian Division, Russian Academy of Sciences, Russia

Example 2: Areas surrounding Greece

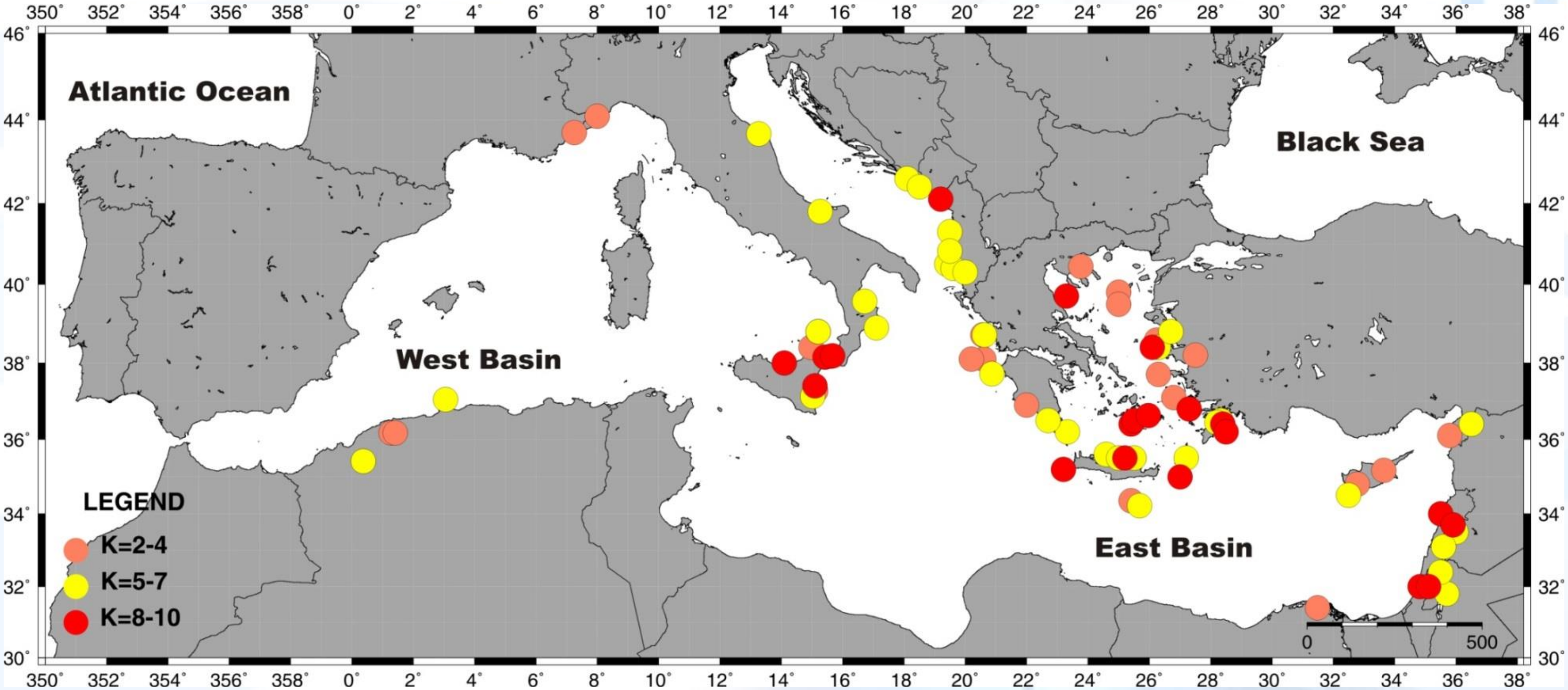
Extreme (historic) and complete events catalogues for

- Hellenic Arc
- Corinth Gulf
- Mediterranean Sea

Tsunami database of Prof G.A. Papadopoulos,
Institute of Geodynamics, National Observatory of Athens, Greece.

Applied Tsunami Catalogue

1620 BC – present – Mediterranean Sea



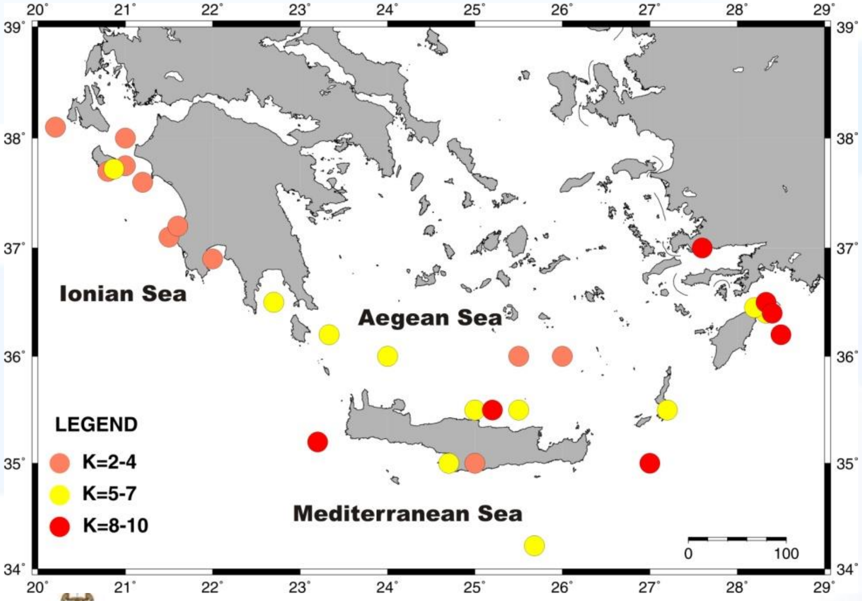
Tsunami database of Prof G.A. Papadopoulos, Institute of Geodynamics, National Observatory of Athens

Applied Tsunami Catalogue

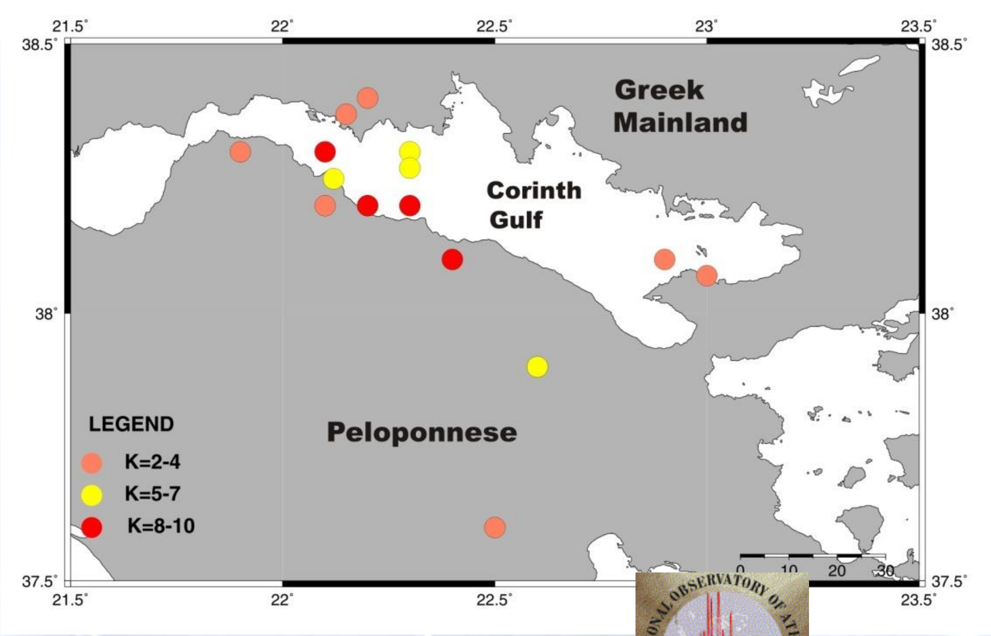
1620 BC – present – Hellenic Arc

373 BC – present – Corinth Gulf

Hellenic Arc



Corinth Gulf



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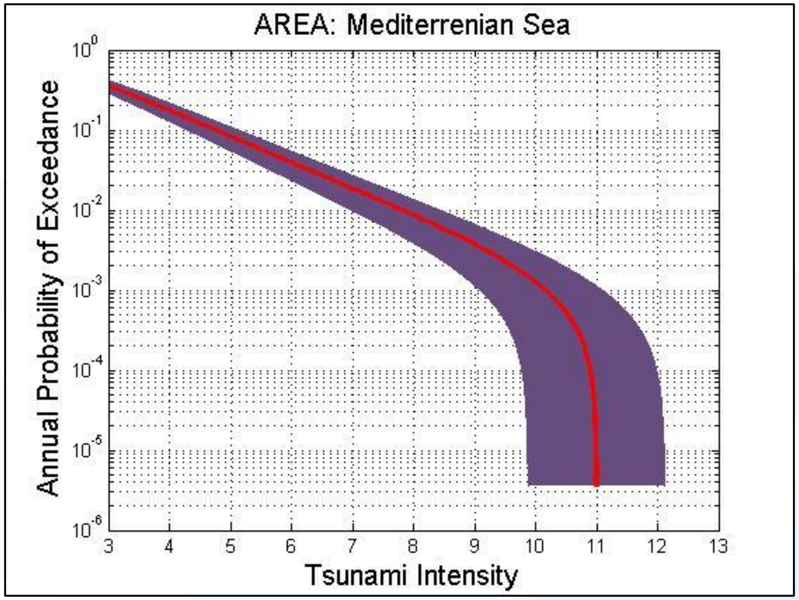
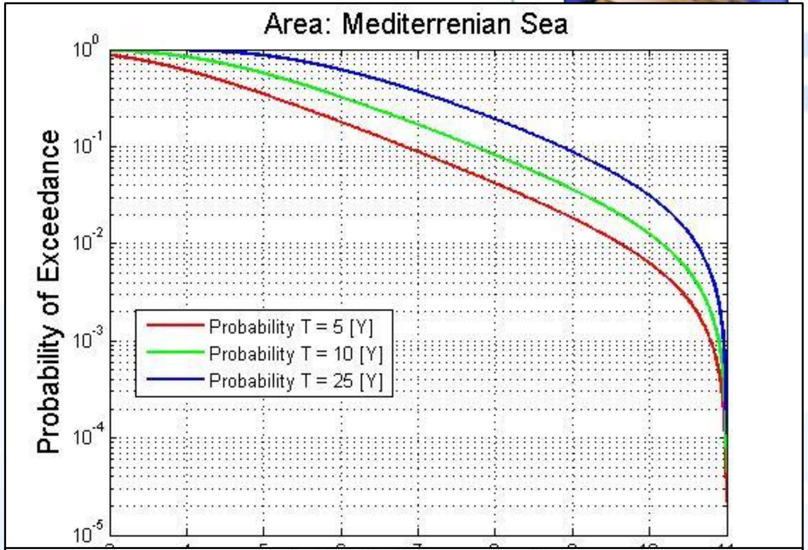
Results: Mediterranean Sea



(a)



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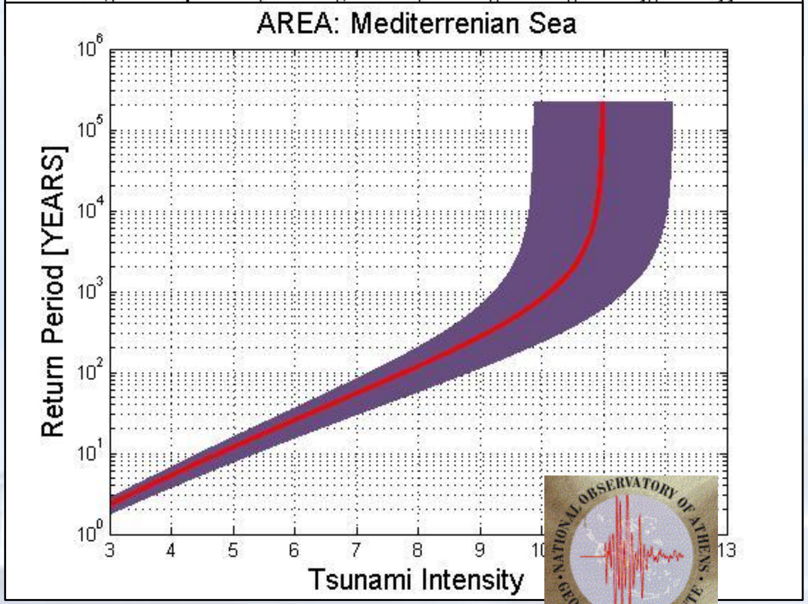


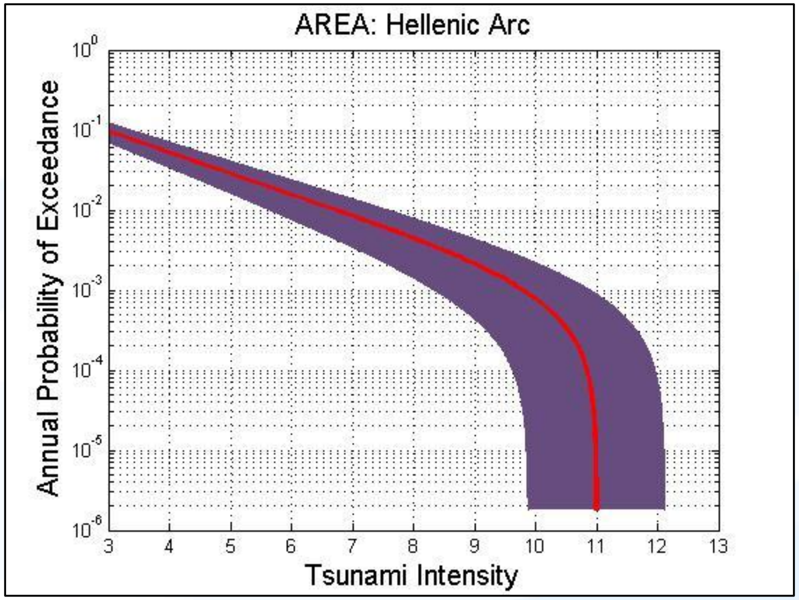
(c)

a. Annual probability of exceedance

b. Probability of exceedance for 5, 10 and 25 yrs

c. Mean return period

Results: Hellenic Arc



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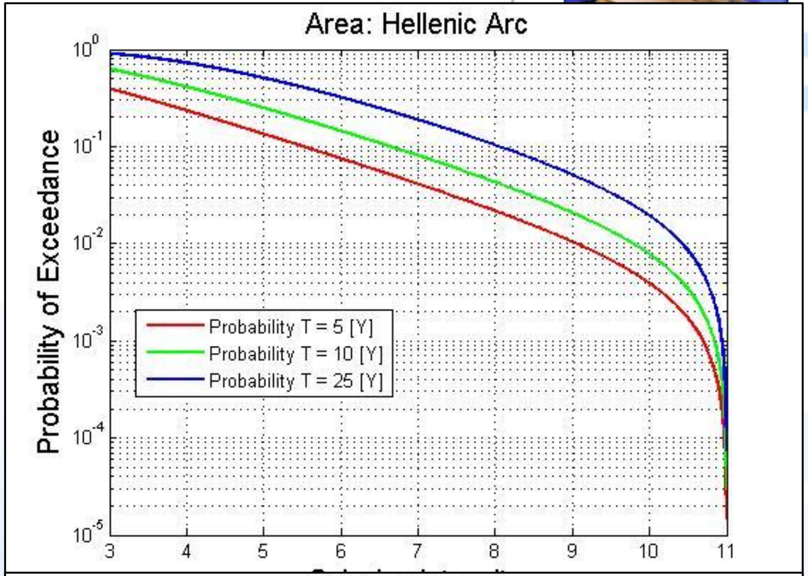

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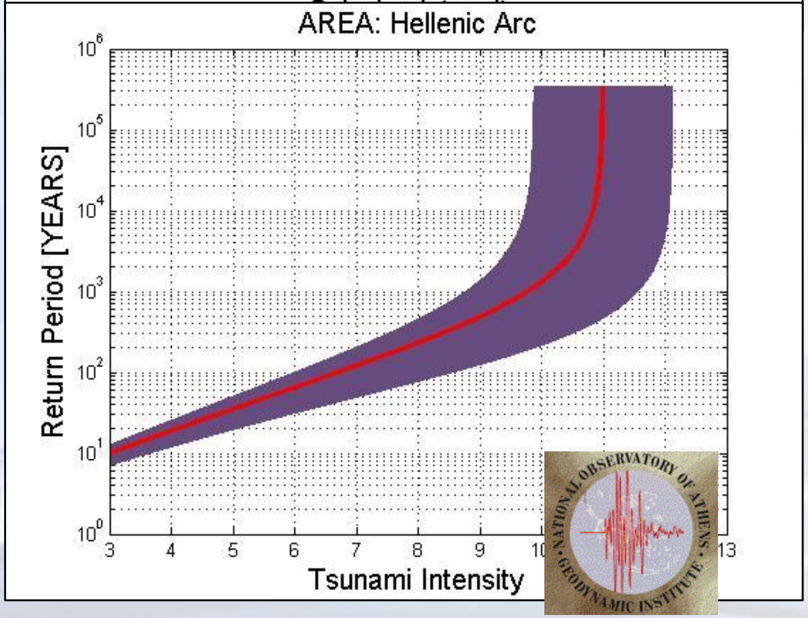

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a. Annual probability of exceedance

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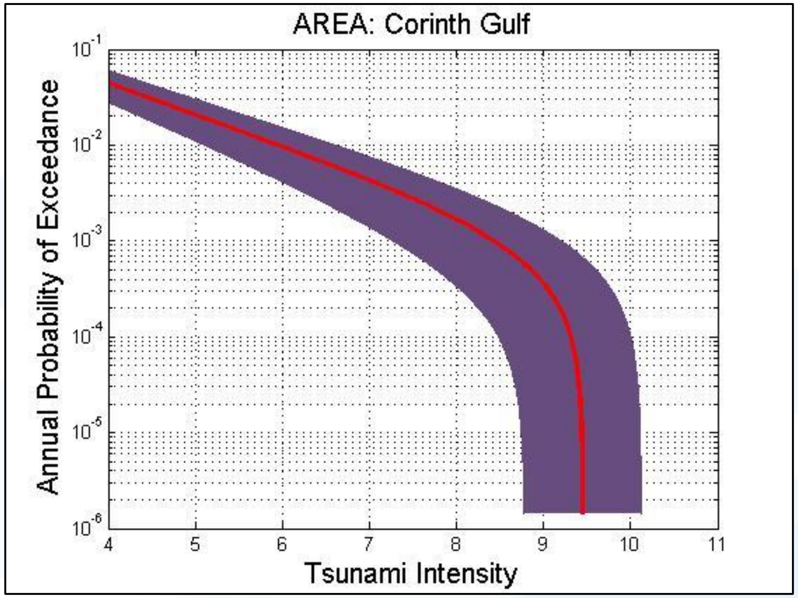
Results: Corinth Gulf



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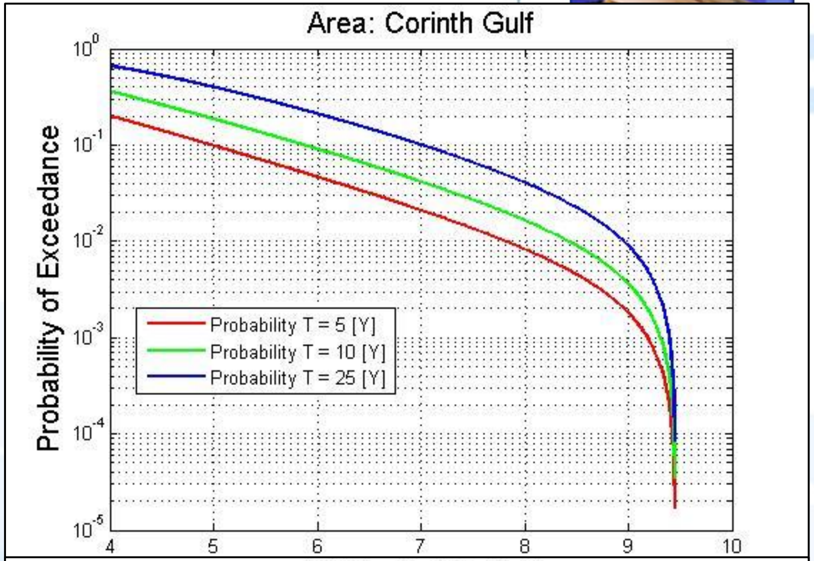

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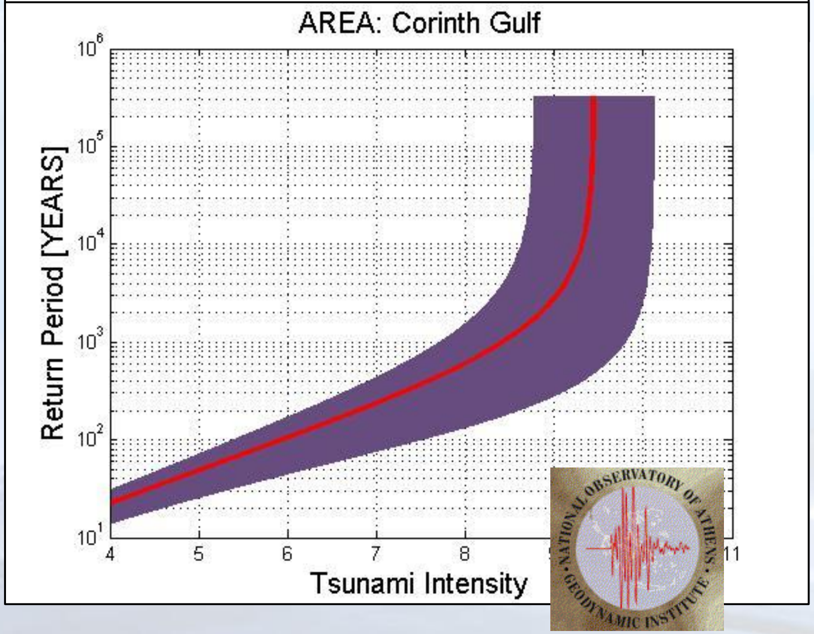

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Mean Return Periods

Return Period	Intensity 5.0	Intensity 9.0
Mediterranean Sea	11.7 yrs	271 yrs
Hellenic Arc	34.6 yrs	477 yrs
Corinth Gulf	48.2 yrs	2800 yrs

Conclusions

- A new procedure for PTHA has been developed to calculate the tsunami hazard for a specified region that caters for **incomplete** and **uncertain** data.
- The procedure:
  - ❖ permits the assessment of the **maximum likelihood** estimates of the key tsunami hazard parameters.
  - ❖ is **flexible** and allows for the use of palaeo, historic and complete tsunami catalogues.
- Do not need an extensive and complete events catalogue

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