Analysis of the temporal properties of Greek aftershock sequences

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Abstract

The temporal distribution of aftershock sequences can be analyzed by means of new parameters other than the Omori’s \( p \)-value. A deep analysis of the temporal properties of six aftershock sequences occurred in Greece from 1980 to 1997 has been carried out by means of fractal tools. We performed the Omori’s law, the R/S and the Allan Factor analyses in order to estimate the \( p \)-value of the aftershock rate, the Hurst exponent (\( H \)) of the interevent-time series, and the spectral exponent (\( \alpha \)) respectively. Our results show a significant variability of the \( p \)-value from 0.89 to 1.48, of \( H \) from 0.78 to 1.186 and of \( \alpha \) from 0.49 to 1.44. The variation of the normalized \( \alpha \) with the threshold magnitude for all the aftershock sequences examined has displayed almost the same decreasing behaviour. The temporal fractality of aftershock sequences is primarily due to the power-law decay of the aftershock rate. © 2001 Published by Elsevier Science B.V.

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1. Introduction

Analysing the temporal properties of seismic sequences is an important step in the general context of studies devoted to seismicity analysis. The time-correlation structures governing observational earthquake time series can give useful information on the dynamical features of seismic processes and on the involved geodynamical mechanisms.

It is well known that seismic processes display a scale-invariant behaviour in several statistical features. Gutenberg and Richter (1944) found that earthquake size follows a power-law distribution. Other scale-invariant features were determined in Kagan (1992) and Kagan and Jackson (1991); Kagan (1994) reviewed experimental evidences for earthquake scale-invariance. A theory to explain the presence of scale-invariance was proposed by Bak et al. (1988) in a pioneer work, by introducing the concept of self-organized criticality (SOC). The analysis of scaling laws concerning earthquakes has led to the development of a wide variety of physical models of seismogenesis (Sornette et al., 1990; Scholz, 1991), used to better characterise the seismicity patterns (Bak and Tang, 1989; Turcotte, 1990; Sornette and Sornette, 1989; Sornette et al., 1991, 1994; Carlson and Langer, 1989; Chen et al., 1991; Ito and Matsuzaki, 1990; Cowie et al., 1993).

Several distributions have been used to model seismic activity. Among these, the Poisson distribu-
tion, which implies the independence of each event from the time elapsed since the previous event, is the most extensively used, since, in many cases, for large events a simple discrete Poisson distribution provides a close fit (Boschi et al., 1995). However, in recent studies it has been shown that earthquake occurrence is characterized by temporal clustering properties with both short and long timescales (Kagan and Jackson, 1991), this implying the presence of time-correlation among the seismic events.

Recently, some authors (Bodri, 1993; Luongo et al., 1996) focused their attention on observational evidence of time clustering properties in earthquake sequences of some different seismic areas, showing the existence of a range of timescales with scaling behaviour. The method they used, the Cantor dust method (Smalley et al., 1987), consists in dividing the total period \((T)\), over which \(N\) earthquakes occur, into a series of \(n\) smaller intervals of length \(t = T/n\) with \(n = 2, 3, 4, \ldots\) and computing the number \(R\) of intervals of length \(t\) which contain at least one event. If the distribution of events has a fractal structure (Smalley et al., 1987), then \(R \sim t^{1-D}\), where \(D\) is the fractal dimension, which has subunitary values: the clustering is higher as \(D\) approaches 0, while a value of 1 corresponds to a uniform distribution (events equally spaced in time). But the parameter \(R\) does not give information about the temporal fluctuations, that are correlated to the power spectral density \(S(f)\) of the process itself, which is power-law shaped for fractal time series, \(S(f) \propto 1/f^a\). An analysis of the temporal fluctuations of seismicity performed by the estimate of its power spectral density is in Bittner et al. (1996) and Lapenna et al. (1998, 2000); but they used the Variance–Time Curve (VTC) method, that for spectra with power-law (fractal) shape furnishes a biased estimate of the spectral exponent depending on the number of samples \(N\), and therefore on the duration of the recording (Thurner et al., 1997; Telesca et al., 2000).

In this paper, we analyze six aftershock sequences that occurred in Greece from 1980 to 1997 in order to study their temporal properties using several methods.

2. Methods

Aftershock sequences, apparently related to the fault plane that slipped during the main event (Lay and Wallace, 1995), are the relevant characteristic of shallow earthquakes. An aftershock series displays temporal fractal properties: the frequency of occurrence of aftershocks decays rapidly following the Omori’s law that states the rate decay of aftershocks series is proportional to \(1/t\), where \(t\) is the lapse time from the mainshock; Utsu et al. (1995) used a modified Omori’s law, \(n(t) = K/(t+c)^p\), where \(K\) and \(c\) are constant and \(p\) is near unity. The Omori’s law gives only a preliminary information of the time-correlation structures that characterises the aftershocks of a main event. A deep investigation of these structures is provided by the analysis of second-order statistics. For this purpose, in this paper the aftershock sequence is modeled by a temporal point process. A temporal point process describes events that occur at some random locations in time (Cox and Isham, 1980), and it can be expressed by a finite sum of Dirac’s \(\delta\) functions centered on the occurrence times \(t_i\).

\[
y(t) = \sum_{i=1}^{n} \delta(t - t_i) .
\]

The process can be represented by the series of interevent time intervals. A preliminary analysis of the temporal properties of a sequence of seismic events can be performed by using the coefficient of variation \(C_V\), defined as \(C_V = \sigma_T/\langle T \rangle\), where \(\langle T \rangle\) is the mean of the interevent times and \(\sigma_T\) is the standard deviation (Kagan and Jackson, 1991): a Poissonian process (completely random) has a \(C_V = 1\), whereas a clustered process is characterized by a \(C_V > 1\). This coefficient does not give information about the timescale ranges where the process can be reliably characterized as a clustered process. Nevertheless, a complex phenomenon can be deeply known only if the different timescales governing its dynamics are well understood. The rescaled range R/S analysis provides information about correlation among blocks of interevent times. For a block of \(m\) interevent times, the difference between each interval and the mean interevent time is calculated and added to a cumulative sum. The normalized range \(R(m)\) is defined as the difference between the maximum and minimum values of the cumulative sum, divided by the standard deviation of the interval size. Fitting \(R(m)\) against \(m\) to the function \(m^{\alpha H}\), where \(H\) is the Hurst exponent (Feder, 1989), we
The Allan Factor (AF) is a measure which can be used to discriminate fractal from Poissonian behaviour in point processes. This factor is related to the variability of successive counts (Allan, 1966; Barnes and Allan, 1966), and it is defined as the variance of successive counts divided by twice the mean number of events in that counting time

\[ \text{AF}(\tau) = \frac{(N_{k+1}(\tau) - N_k(\tau))^2}{2\langle N_k(\tau) \rangle} . \]  

The AF of a fractal point process varies with the counting time \( \tau \) with a power-law form

\[ \text{AF}(\tau) = 1 + \left( \frac{\tau}{\tau_1} \right)^\alpha . \]  

The monotonic power-law increase is representative of the presence of fluctuations on many time scales (Lowen and Teich, 1995); \( \tau_1 \) is the so-called fractal onset time, and marks the lower limit for significant scaling behaviour in the AF (Teich et al., 1996), so that for \( \tau < \tau_1 \) the clustering property becomes negligible within these timescales. For Poissonian processes, the AF is always near or below unity for all counting times \( \tau \). From Eq. (4), the estimate of \( \alpha \) can be performed plotting the AF in bilogarithmic scales, and then calculating the slope of the straight line, that fits AF in its linear range by using a least squares method. The exponent \( \alpha \) conveys the information about the temporal fluctuations of the process, because it is the exponent of the power spectrum that for fractal processes decays as a power-law function of the frequency \( f \), \( S(f) \propto f^{-\alpha} \), with \( \alpha \) measuring the strength of the clustering. Of course, for a finite size fractal real process the power spectral density behaves as an inverse power-law function in a limited range of frequency, approaching an asymptotic value at high frequencies, at which the behaviour of the process can be considered Poissonian. The numerical value of \( \alpha \) is an indicator of the presence of clusterization in the process (Thurner et al., 1997). If the point process is Poissonian, the occurrence times are uncorrelated; for this memoryless process \( \alpha \approx 0 \). On the other side, \( \alpha \neq 0 \) is typical of point processes with scaling behaviour.

3. Geological and seismological settings

The region of the Aegean sea and the surrounding areas in the Eastern Mediterranean lies on the boundary zone between the Eurasian and the African plates (Mckenzie, 1970). The region is a zone of widespread deformation within which complex relationships exist between extensional, compressional and strike–slip deformations (Mckenzie, 1978; Dewey and Sengor, 1979; Le Pichon and Angelier, 1979). As expected in an area of extensive deformation, this reveals high seismic activity, the highest in Europe. Most of the seismicity is associated with the Hellenic arc and the fault zones of Western and Northern Anatolia (Papazachos, 1973). The Hellenic arc shows remarkable features of Island arc such as the existence of intermediate depth earthquakes and volcanic activity. The African plate is moving northeastward relative to the Aegean plate and is subducted beneath the western

Subduction of the Ionian lithosphere, supposed of continental type (Mckenzie, 1978; Le Pichon and Angelier, 1979) occurs westward beneath the Calabrian arc and eastward beneath the Hellenic and Dinaric Alps. Extensional processes and consequent crustal stretching, presumably started in the Miocene, involve the Tyrrenhian and Aegean back arc basins (Moretti and Royden, 1988).

The extensional tectonics in the Aegean basin has not yet led to the creation of lithosphere of oceanic type. Moreover, the convergence of the Eurasian and the African plates has induced crustal shortening and thickening of Hellenides chains. Both the Ionian and the Aegean region are bordered with belts of seismic activity more intense in comparison to surrounding zones. Seismicity is associated with the orogenesis of Hellenides belt, which shows remarkable features of island arcs: volcanic activity (Fytikas et al., 1985) and intermediate depths earthquakes up to 200 km in the Aegean region (Mckenzie, 1978). Recent studies on the upper mantle (Drakatos, 1989; Drakatos and Drakopoulos, 1991; Spakman et al., 1993; De Jonge et al., 1994; Alessandrini et al., 1997; Drakatos et al., 1997) show the presence of a high velocity anomaly dipping northeast beneath the Aegean sea down to a depth of at least 600 km.

Other geophysical observations also show that the Aegean region is in an anomalous tectonic setting. Heat flow data indicate high heat flow with a mean value of 2.1 HFU in the northern and central Aegean sea (Fytikas et al., 1985). A compilation of gravity data shows a large positive gravity anomaly in the Aegean sea region, while negative anomalies are observed in Turkey and in the mainland of Greece (Lagios et al., 1995).

The data set consists of six aftershock sequences which occurred in Greece from 1980 to 1997 (Fig. 1). The aftershock lists are compiled by Latoussakis and Drakatos (1994), Drakatos and Latoussakis (1996) and Drakatos (2000) based on the data published in the Bulletins of the Institute of Geodynamics (National Observatory of Athens). According to them (i) in all cases the magnitude of the mainshock is $M_L > 5.0$, while the minimum magnitude of the aftershock is $M_L = 3.0$, this is the magnitude to which the catalog is complete for the whole area and time period; (ii) the duration of the aftershock activity was considered in comparison to the rate of the background seismicity of each region; (iii) aftershocks are sampled thoroughly from the whole source region of the mainshock.

4. Analysis of the aftershock sequences

For each aftershock series, we performed the following analyses: the Gutenberg–Richter analysis to estimate the parameters of the power-law magnitude–frequency relation; the modified Omori’s law fitting in order to estimate the $p$-value; the calculation of the coefficient of variation to get preliminary information about the clustering properties; the R/S analysis to estimate the Hurst exponent and its relation with the $z$ exponent; the Allan Factor analysis in order to estimate the $z$ exponent. We repeated the Allan Factor analysis with increasing threshold magnitude in order to evidence possible changes of the scaling behaviour with magnitude. In all the calculations we ranged the counting time from $T_{10^5}$ s to $T_{10}$, where $T$ is the total period of observation of the sequence. Furthermore, we performed the Hurst analysis on the aftershock sequences increasing the threshold magnitude. Varying the threshold magnitude, we stopped the analysis at a threshold magnitude for which the number of events were not lower than about 50 events. We fitted by the least-squares method the cumulative Gutenberg–Richter law by a straight line, whose slope has been estimated to give the $b$-value (Shi and Bolt, 1982). Since estimate of Omori’s $p$-value depends on the length of unit time interval for the rate decay $n(t)$, the best estimation procedure is that directly based on the time series rather than on the discrete distribution of the number of aftershocks per unit time interval (Ogata, 1983). For this reason, to obtain the estimate of the $p$-value, we applied a nonlinear fitting procedure (Jiménez and García-Fernández, 1996) to the cumulative number

$$N(t) = k \cdot \left\{ \frac{(t+c)^{1-p} - c^{1-p}}{1-p}, \quad p \neq 1 \right\} \ln(t + c) - \ln(c), \quad p = 1$$

that represents the integral of the modified Omori’s law (Utsu et al., 1995).
Several papers discussed the relation between the p-value, the b-value and the fractal dimension D of the hypocenters of sequences of aftershocks, obtaining different conclusions (positive, negative, and no-correlation) (Guo and Ogata, 1997). In this paper, we use new parameters other than p to characterize the temporal fluctuations of an aftershock sequence.

4.1. The M=6.0 July 9, 1980 earthquake

The length of the sequence is almost 1.2 × 10^7 s (July 9, 1980–November 22, 1980) and contains 189 aftershocks with magnitude M ≥ 3.2. The Gutenberg–Richter analysis (Fig. 2a) gives b = 0.84 ± 0.05. The coefficient of variation, evaluated by the interevent–interval analysis (Fig. 2b) is CV ≈ 3.5. The modified Omori’s law fit has furnished p = 1.48 ± 0.07 (Fig. 2c). We performed the R/S analysis of the interevent–intervals (Fig. 2d), and we recognized the presence of linear behaviour, with an estimate of the Hurst exponent H = 0.78 ± 0.01, and consequently, α = 0.56 ± 0.08.

The AF (Fig. 2e) is characterized by an increasing linear trend, thus indicating the existence of clusters over long timescales, but presents two drops at the timescales of about 10^5 and 10^6 s. The underlying linear trend seems to begin at T1 = 1.2 × 10^4 s, with α = 0.49 ± 0.02, close to the estimate obtained by the R/S method.

Increasing the threshold magnitude Mth (Fig. 2f), the AF curves behave quite similarly, although in the long-timescale range, poorer statistics, due to the decreased number of events, can influence the behavioural trend of the curves.
4.2. The $M=6.3$ February 24, 1981 earthquake

The length of the sequence is almost $1.6 \times 10^7$ s (February 24, 1981–August 27, 1981) and contains 539 aftershocks with magnitude $M \geq 3.2$. Analysing the frequency–magnitude relation (Fig. 3a), we estimate $b = 0.87 \pm 0.04$. The coefficient of variation (Fig. 3b) is $C_V \approx 3.1$. The modified Omori’s law fit fur-
nishes $p = 1.25 \pm 0.03$ (Fig. 3c). The R/S analysis of the interevent–intervals (Fig. 3d) shows a monotonically increasing behaviour with $H = 1.01 \pm 0.01$, giving $\alpha = 1.02 \pm 0.02$.

The AF (Fig. 3e) shows an increasing linear trend, with one sensible drop at the timescale of almost $4 \cdot 10^5$ s. The linear trend seems to start at $T_1 = 4 \cdot 10^3$ s, with $\alpha = 0.73 \pm 0.04$. 

Fig. 3. Aftershocks of February 24, 1981 earthquake. (a) Gutenberg–Richter relation. (b) Interevent times distribution. (c) Omori’s law fitting procedure. (d) R/S analysis. (e) Allan Factor analysis for the set of events with magnitude $M \geq 3.2$. (f) Allan Factor curves with threshold magnitude ranging from 3.2 to 4.2.
Varying the threshold magnitude $M_{th}$ (Fig. 3f), the AF curves show the same functional dependence on the timescale, with shortening dynamics increasing the threshold.

4.3. The M=6.3 December 19, 1981 earthquake

The length of the sequence is almost $2.2 \times 10^7$ s (December 19, 1981 – August 31, 1982) and contains
278 aftershocks with magnitude \( M \geq 3.2 \). Analysing the frequency–magnitude relation (Fig. 4a), we estimate \( b = 0.73 \pm 0.06 \). The coefficient of variation (Fig. 4b) is \( CV \approx 2.3 \). Fitting with the modified Omori’s law, we estimate \( p = 0.944 \pm 0.006 \) (Fig. 4c). The R/S analysis of the interevent–intervals (Fig. 4d) allows to...
evaluate the Hurst exponent relatively $H = 0.97 \pm 0.01$, and as a consequence, the value of $z$ of approximately 0.94.

The AF (Fig. 4c) shows an increasing linear trend similar to that observed for the July 9, 1980 sequence, with three drops at the timescales of about $1.3 \cdot 10^4$,
5.10^4 and 5.2.10^5 s. The supporting linear trend seems to begin at \( T_1 = 4.10^3 \) s, with \( \alpha = 0.63 \pm 0.01 \), quite near to R/S measure.

The variation with the threshold magnitude \( M_{th} \) (Fig. 4f) shows that the AF curves are only translated, but the behavioural trend still remains.

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Fig. 7. Aftershocks of November 18, 1997 earthquake. (a) Gutenberg–Richter relation. (b) Interevent times distribution. (c) Omori’s law fitting procedure. (d) R/S analysis. (e) Allan Factor analysis for the set of events with magnitude \( M \geq 3.0 \). (f) Allan Factor curves with threshold magnitude ranging from 3.0 to 3.9.
4.4. The M=6.2 January 17, 1983 earthquake

The length of the sequence is almost $5.6 \times 10^6$ s (January 17, 1983–June 21, 1983) and contains 267 aftershocks with magnitude $M \geq 3.7$. Performing the Gutenberg–Richter analysis (Fig. 5a), we obtained a $b$-value of $0.91 \pm 0.08$ for the cumulative distribution of events with magnitude $M \geq 3.7$. From the interevent–interval series (Fig. 5b), we calculated the coefficient of variation $C_V \approx 2.5$. Fitting the data with the modified Omori’s law, we obtained $p = 1.03 \pm 0.01$ (Fig. 5c). We performed the R/S analysis on the interevent time data set, obtaining $H = 1.04 \pm 0.01$, and therefore, $\alpha = 1.08 \pm 0.01$ (Fig. 5d).

The AF plot (Fig. 5e) gives clear indication of the fractal behaviour; the AF is seen to increase with linear behaviour for counting times greater than approximately $T_1 = 4.4 \times 10^4$ s in bilogarithmic scales. We estimated $\alpha = 1.44 \pm 0.04$ for events with $M \geq 3.7$. This value is in good accordance with the measure of the spectral exponent obtained by the R/S method.

4.5. The M=6.1 May 13, 1995 earthquake

The length of the sequence is almost $2.7 \times 10^7$ s (May 13, 1995–March 24 1996) and contains 633 aftershocks with magnitude $M \geq 3.0$. The Gutenberg–Richter analysis (Fig. 6a) furnishes $b = 1.08 \pm 0.06$. The coefficient of variation, evaluated by the interevent–interval analysis (Fig. 6b) is $C_V \approx 3.6$. The modified Omori’s law fit has given $p = 1.411 \pm 0.006$ (Fig. 6c). The R/S analysis of the interevent–interval series (Fig. 6d) furnished the $H$-exponent estimate, with $H = 1.186 \pm 0.007$ and $\alpha = 1.37 \pm 0.01$.

5. Discussion

In Table 1, we summarize the parameters that characterize the aftershock time-dynamics. The $b$-
The correlation coefficient among the parameters above mentioned. The $b$-value seems to be well correlated with the $x$-exponent. The coefficient of variation is strongly correlated with the $p$-value. The correlation of $H$ with $x$ is obvious, because of the relation between these two parameters.

The six aftershock sequences can be distinguished in approximately two groups: group A (January 17, 1983; November 18, 1997 and May 13, 1995) and group B (July 9, 1980; December 19, 1981 and February 24, 1981).

The group A is characterized by a $b$-value near to unity; furthermore, the spectral exponent $x$ assumes a value greater than 1 for all the three series. We observe also a similar value for the fractal onset time $T_1 \approx 10^3$ s, that is the cutoff time, starting from which a linear behaviour in the AF variation is detectable.

For each aftershock sequence, we calculated the $x$-values varying the threshold magnitude, and in order to compare the variations among the different aftershock series we normalized the obtained values. We observe (Fig. 8) that the variation of $x$ with the threshold magnitude behaves approximately in the

![Fig. 8](image)

Fig. 8. Normalized $x$-exponent evaluated for all the six aftershock sequences.

![Fig. 9](image)

Fig. 9. Normalized $H$-values evaluated for all the six aftershock sequences.

![Fig. 10](image)

Fig. 10. Scatter-plot $p-C_V$ related to the randomly generated aftershock sequences for four different values of $p=0.8, 0.9, 1.0$ and 1.1.

The group B is characterized by a relatively low $b$-value; the exponent $x$ is well below unity and also the fractal onset time $T_1$ takes lower values (from $10^3$ to $10^4$ s).

For each aftershock sequence, we calculated the $x$-values varying the threshold magnitude, and in order to compare the variations among the different aftershock series we normalized the obtained values. We observe (Fig. 8) that the variation of $x$ with the threshold magnitude behaves approximately in the
same manner in all the cases examined. In order to analyse the magnitude-dependence of the Hurst exponent $H$, we calculated the normalized $H$-value for each aftershock sequence. Fig. 9 shows the results that present a clear decrease of $H$ with the threshold magnitude, consistent with the behaviour of the normalized $\alpha$.

In order to check the validity of the methods described in the paper, we generated several series of aftershocks, following in time the Omori and in magnitude the Gutenberg–Richter distribution. The series have been generated with $b=0.8$, $M_0=3.0$ and $p=0.8$, 0.9, 1.0 and 1.1. For each series, we estimated the coefficient of variation $C_V$, the Hurst exponent $H$ and the $p$-value. Figs. 10, 11 and 12 show the $p-C_V$, the $p-H$ and $H-C_V$ scatter-plots, respectively. The correlation between $p$ and $C_V$ is very good ($r=0.96$), as in the observational cases. Furthermore, we performed the Allan Factor analysis on the aftershock sequences, randomly generated with $p=0.8$. We repeated the analysis varying the threshold magnitude. In Fig. 13, the estimates of $\alpha$ for each random sequence are presented with increasing the threshold magnitude; in Fig. 14 we also show the variation of the mean normalized $\alpha$. We observe the decrease of $\alpha$ with increasing the threshold, this confirming the results obtained analyzing the real cases.

**Fig. 11.** Scatter-plot $p-H$ related to the randomly generated aftershock sequences for four different values of $p=0.8$, 0.9, 1.0 and 1.1.

**Fig. 12.** Scatter-plot $H-C_V$ related to the randomly generated aftershock sequences for four different values of $p=0.8$, 0.9, 1.0 and 1.1.

**Fig. 13.** Normalized $\alpha$ versus threshold magnitude for the randomly generated aftershock sequences with $p=0.8$.

**Fig. 14.** Mean normalized $\alpha$ versus threshold magnitude for the randomly generated aftershock sequences with $p=0.8$. 
6. Conclusions

The sequences of aftershocks of six earthquakes that occurred in Greece from 1980 to 1997 have been studied in order to examine their temporal properties. We performed different analyses: the Omori’s law fit has furnished the values of $p$ that represents the scaling coefficient of the aftershock rate; the R/S analysis has given the values of the Hurst exponent that characterizes the persistence of the time series; the Allan Factor method has given the estimates of the $\alpha$-exponent, that is the scaling coefficient of the power spectral density and that reveals the presence of clustering behaviour in the aftershock processes. The observational results have been confirmed by the numerical investigation. The performed analyses have discriminated two groups in the aftershock series set with similar scaling characteristics.

References


